INFLUENCE OF STRESS FIELD NONSTATIONARITY ON CRACK GROWTH UNDER CREEP

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More and more attention is paid to investigations of the regularities of crack growth in metals under high-temperature creep conditions at this time. These are both experimental papers on the clarification of parameters governing the rate of crack growth in specimens (see [1-3], etc.), and theoretical papers examining the problem of crack growth under different initial assumptions about the stress distribution near the crack apex and the crack propagation criteria (see [4-10], etc.).

The process of stress redistribution from the initial elastic state to the steady creep state that occurs in a solid with a crack after the load application is not taken into account in an absolute majority of theoretical papers. A solution of such a problem is represented in [11] for a power law of creep with hardening and softening of the material taken into account. A finite-element analysis of the stress field, performed in [12], verified the validity of the asymptotics obtained in [11].

1. The fundamental relationships of [11] for the plane strain case are these: a) at the initial instant ($t = 0$), the stress field has the form

$$\sigma_{ij}(r, \theta, t=0) = \frac{K_I}{\sqrt{2\pi r}} \tilde{\sigma}_{ij}^{(e)}(\theta); \tag{1.1}$$

b) for $t > 0$ a domain expanding with the lapse of time originates in direct proximity to the crack apex, and creep strains predominate therein, while the stress field there has the same form as in [13, 14]:

$$\sigma_{ij}(r, \theta, t) = \left(\frac{C(t)}{BI_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}^{(c)}(n, \theta); \tag{1.2}$$

while an elastic stress distribution (1.1) is conserved outside this domain. Here r, θ are polar coordinates; $\tilde{\sigma}_{ij}^{(e)}(\theta)$, $\tilde{\sigma}_{ij}^{(c)}(n, \theta)$, known functions of the angular variation in the stress in the elastic state and in the state of steady creep; B, n, constants in the power law of creep $\dot{\varepsilon}^{(c)} = B\sigma^n$; I_n, a known constant [13],

$$C(t) = \begin{cases} C_* t_T/t, & 0 < t < t_T, \\ C_*, & t \geqslant t_T; \end{cases}$$

C_* is the steady creep integral independent of the contour analogous to the J-integral of Cherepanov–Rice [2], $t_T = (1 - \nu^2) \times K_I^2/[(n + 1)EC_*]$ is the build-up time of the stationary state of steady creep near the crack apex. Domains of predominant creep strain and elastic strain are separated by a certain transition zone. Neglecting this zone, we find from the condition of continuity of the normal stress on the boundary separating the domain of predominant creep strain (the asymptotic (1.2)) and of elastic strain (the asymptotic (1.1)), the equation of the line separating these domains

$$r_1(\theta, t) = \left(\frac{K_I^2}{2\pi}\right)^{\frac{n+1}{n-1}} \left(\frac{BI_n}{C(t)}\right)^{\frac{2}{n-1}} F(\theta), \tag{1.3}$$

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where $F(\theta)$ is an unknown function of the polar angle θ defined by $\tilde{\sigma}_{ij}^{(e)}(\theta)$ and $\tilde{\sigma}_{ij}^{(c)}(n, \theta)$ and such that $F(0) = 1$. The legitimacy of such a stress field approximation (the localization of the transition zone at the line (1.3)) is confirmed by results in [15].

2. The damage parameter $\omega(x, t)$ $(0 \leqslant \omega \leqslant 1)$ with its kinetic equation [16]

$$\frac{\partial\omega}{\partial t} = A\left(\frac{\sigma_{\max}}{1-\omega}\right)^m \tag{2.1}$$

was used in [7] to describe crack growth under creep. The dependence

$$1-(1-\omega(x, t))^{m+1} = A(m+1)\int_0^t \sigma_{\max}^m(x, \tau)\,d\tau \tag{2.2}$$

is found for $\omega(x, t)$ from (2.1). Having selected the condition $\omega(l(t), t) = 1$ as the crack propagation criterion (the damage at the crack apex has its limit value $\omega = 1$) we obtain an equation to find the law of crack growth during creep

$$1 = A(m+1)\int_0^t \sigma_{\max}^m(l(t), \tau)\,d\tau. \tag{2.3}$$

For $\sigma_{\max} = \sigma_y$ in conformity with the results in [11] we select the following approximation

$$\sigma_{\max}(x, t) = \begin{cases} \left(\dfrac{C(t)}{BI_n}\dfrac{1}{x-l(t)}\right)^{\frac{1}{n+1}}, & 0 < x - l(t) < R(t), \\ \dfrac{K_I}{\sqrt{2\pi(x-l(t))}}, & x - l(t) \geqslant R(t), \end{cases} \tag{2.4}$$

where $R(t) = r_1(0, t)$. Substituting (2.4) into (2.3), we convert it to the form

$$\frac{1}{A(m+1)} = \int_0^{\tau(t)} \left(\frac{K_I^2}{2\pi}\right)^{\frac{m}{2}} \frac{d\tau}{(l(t)-l(\tau))^{\frac{m}{2}}} + \int_{\tau(t)}^{t} \left(\frac{C(\tau)}{BI_n}\right)^{\frac{m}{n+1}} \frac{d\tau}{(l(t)-l(\tau))^{\frac{m}{n+1}}}, \tag{2.5}$$

where $\tau(t)$ is the solution of the equation $l(\tau) + R(\tau) = l(t)$. Let us use the notation $m/(n+1) = \alpha$, $0 < \alpha < 1$, $m/2 = \beta$, $0 < \beta$, $l(t) - l_0 = z$ and let us introduce the new variable $\xi = l(\tau) - l_0$. Then (2.5) is written in the form

$$\frac{1}{A(m+1)} = \int_0^{\xi(z)} \left(\frac{K_I^2}{2\pi}\right)^{\beta} \frac{\tau'(\xi)\,d\xi}{(z-\xi)^{\beta}} + \int_{\xi(z)}^{z} \left(\frac{C(\xi)}{BI_n}\right)^{\alpha} \frac{\tau'(\xi)\,d\xi}{(z-\xi)^{\alpha}}. \tag{2.6}$$

Here $\xi(z)$ is the solution of the equation $\xi + R(\xi) = z$.

To seek the solution of Eq. (2.6), to determine the unknown dependence $\tau(\xi)$, we apply the Laplace transform in the variable z. After a change in the order of integration in the right side, Eq. (2.6) can be reduced to the form

$$\frac{1}{A(m+1)p} = \int_0^\infty e^{-p\xi}\tau'(\xi)\left\{\left(\frac{K_I^2}{2\pi}\right)^{\beta}\int_R^\infty t^{-\beta}e^{-pt}dt + \left(\frac{C(\xi)}{BI_n}\right)^{\alpha}\int_0^R t^{-\alpha}e^{-pt}dt\right\}d\xi. \tag{2.7}$$

Let us write the asymptotics for the integrals in (2.7) corresponding to the case of large p:

$$\begin{aligned} &\int_R^\infty t^{-\beta}e^{-pt}dt = \frac{e^{-pR}R^{1-\beta}}{pR}\left(1-\frac{\beta}{pR}+O\left(\left(\frac{1}{pR}\right)^2\right)\right), \\ &\int_0^R t^{-\alpha}e^{-pt}dt = \frac{\Gamma(1-\alpha)}{p^{1-\alpha}} - \frac{e^{-pR}R^{1-\alpha}}{pR}\left(1-\frac{\alpha}{pR}+O\left(\left(\frac{1}{pR}\right)^2\right)\right). \end{aligned} \tag{2.8}$$

Taking account of asymptotic (2.8), Eq. (2.7) has the form

$$\frac{1}{A(m+1)p} = \frac{\Gamma(1-\alpha)}{p^{1-\alpha}} \int_0^\infty e^{-p\xi}\tau'(\xi)\left(\frac{C(\xi)}{BI_n}\right)^\alpha \left\{1 + \frac{(\alpha-\beta)e^{-pR}}{\Gamma(1-\alpha)(pR)^{1+\alpha}}\left(1+O\left(\frac{1}{pR}\right)\right)\right\} d\xi. \tag{2.9}$$

Let $\varphi_1(p) = \int_0^\infty e^{-p\xi}\tau'(\xi)\left(\frac{C(\xi)}{BI_n}\right)^\alpha d\xi$ and $\varphi_2(p) = \int_0^\infty e^{-pz}\tau'(z)\left(\frac{C(z)}{BI_n}\right)^\alpha \frac{dz}{R(z)^{1+\alpha}}$ (the change of variable $z = \xi + R(\xi)$) has been made in the second integral). Then we can write for (2.9)

$$\frac{1}{A(m+1)p} = \frac{\Gamma(1-\alpha)}{p^{1-\alpha}}\varphi_1(p) + (\alpha-\beta)\frac{\varphi_2(p)}{p^2}\left(1+O\left(\frac{1}{p}\right)\right). \tag{2.10}$$

For large p the solution of (2.10) has the form

$$\varphi_1(p) = \frac{1}{\Gamma(1-\alpha)A(m+1)p^\alpha}\left(1+O\left(\frac{1}{p}\right)\right). \tag{2.11}$$

Going from the transform to the original in (2.11), we obtain

$$\tau'(z)\left(\frac{C(z)}{BI_n}\right)^\alpha = \frac{z^{\alpha-1}}{\Gamma(1-\alpha)\Gamma(\alpha)A(m+1)}(1+o(z)). \tag{2.12}$$

For small z the rate of crack growth can be found from (2.12)

$$\dot{l} = \frac{dz}{dt} = A(m+1)\Gamma(\alpha)\Gamma(1-\alpha)\left(\frac{C(z)}{BI_n}\right)^\alpha z^{1-\alpha}. \tag{2.13}$$

For $t < t_T$ the relationship (2.13) takes the form

$$\dot{l} = A(m+1)\Gamma(\alpha)\Gamma(1-\alpha)\left(\frac{(1-\nu^2)K_I^2}{(n+1)I_nEBt}\right)^\alpha (l-l_0)^{1-\alpha}. \tag{2.14}$$

Integrating (2.14) and getting rid of t, the relationship for the rate of crack growth $\dot{l}$ for $t < t_T$ can be written in the form

$$\dot{l} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(A(m+1)\Gamma(\alpha)\Gamma(1-\alpha)\left(\frac{1-\nu^2}{(n+1)I_nEB}\right)^\alpha\right)^{\frac{1}{1-\alpha}} K_I^{\frac{2\alpha}{1-\alpha}}(l-l_0)^{1-\frac{\alpha}{1-\alpha}}.$$

Therefore, in the nonstationary stage of the stress redistribution from the initial elastic state to the steady creep state the rate of crack growth is determined by the stress intensity factor K_I. An experimental dependence of the rate of crack growth on K_I was observed in a number of experiments [1, 3, 8-10].

For $t > t_T$ relationship (2.13) takes the form

$$\dot{l} = A(m+1)\Gamma(\alpha)\Gamma(1-\alpha)\left(\frac{C_*}{BI_n}\right)^\alpha (l-l_0)^{1-\alpha},$$

i.e., the rate of crack growth is already determined by the C_*-integral. The experimental dependence $\dot{l}(C_*)$ was observed in [2, 3, 8-10]. Such a transition in the parameters governing the crack growth rate was also observed in a number of experiments when some authors obtained the dependence $\dot{l}(K_I)$ and others $\dot{l}(C_*)$ in the very same material [3, 8-10]. This can apparently be related to the specimens on which the experiments were performed (the kind of specimen affects the quantity K_I and C_*, while they in turn affect the quantity t_T), or to the time interval in which the experiment was performed (the time of performing the experiment t is less than or greater than t_T).

The deduction can be made from the solution presented above for the crack growth, that the stress redistribution under creep can substantially affect the parameter governing the crack growth rate.

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